Proportional - metrological and geometric peculiarities in the logic of Roman Pantheon design

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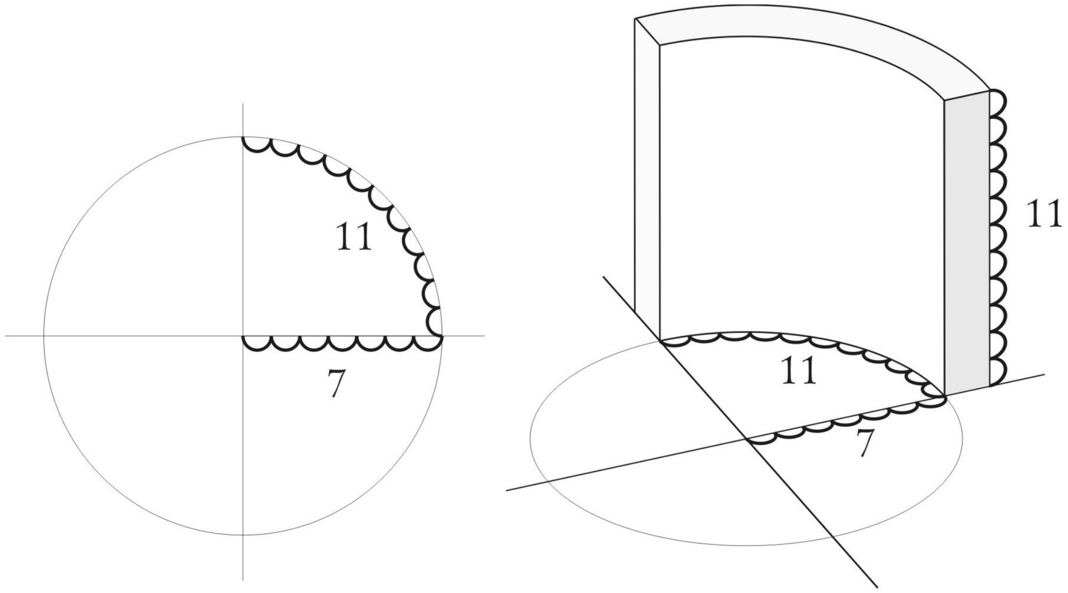
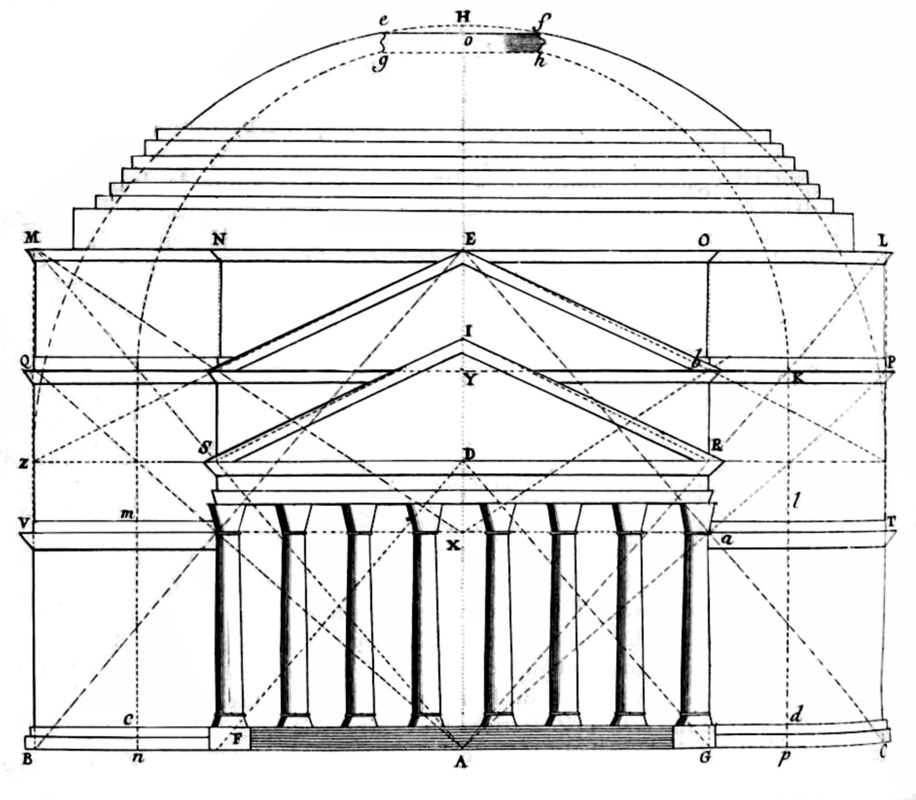
The research history of the Roman Pantheon forms is as unique as the place which this monument occupies in the history of world architecture. We can see it mentioned in the treatises of many theorists of architecture such as: Alberti, Palladio, Serlio, Fontano and the others. In the fourth book of his treatise Palladio gives ten very detailed drawings providing a great number of sizes which he evaluated using the foot of Vicenza (0,35m). It should be highlighted that while fixing forms of the Pantheon in details, Palladio didn't try to reveal any logic of shaping and to define any proportional laws. The same approach could be seen in the works of Serlio and Fontano who just mentioned various sizes of the Pantheon, using the Roman linear measures (0,22м). However, in the Alberti's treatise there is an observation containing some attempt to understand the logic of designing the Pantheon. While describing round constructions including the Pantheon, he explains that the most skilled architects divided the area of a round site into four parts, then they straightened one of the lines and according to its length they erected an internal wall, at a ratio of eleven to fourteen (VII, 10). The Russian researcher V. P. Zubov commenting on this fragment of the treatise appendix, believes that it is an integer substitute of the irrational number «Pi», which was expressed by the fraction 22/7 [1] (pic. 1).



Pic.1. The scheme of construction of building circular in plan according to Alberti.

The repeated measurements of the Pantheon were published in the end of the 17th century by the French researcher Antoine Desgodet[2] who used the so-called Parisian foot (0,324m) for this purpose. Simultaneously with the French type of measures, Desgodet also used small linear measures called "parts" which are equal to 1/30 radius of the columns.

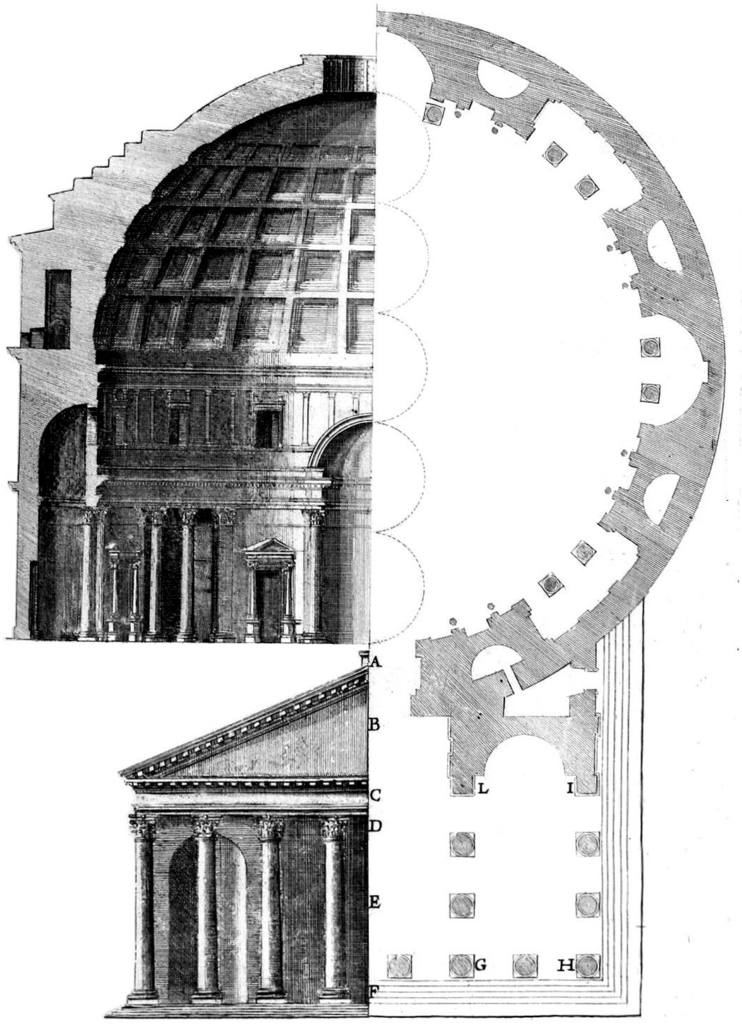
Almost at the same time a treatise by Francois Blondel (1618-1686) was published in Paris. Francois Blondel was, most likely, the first researcher who decided to define a logic behind the shaping of architectural landmarks including the Pantheon (pic. 2).



Pic.2. The scheme of the proportional analysis of the Pantheon facade according to Blondel.

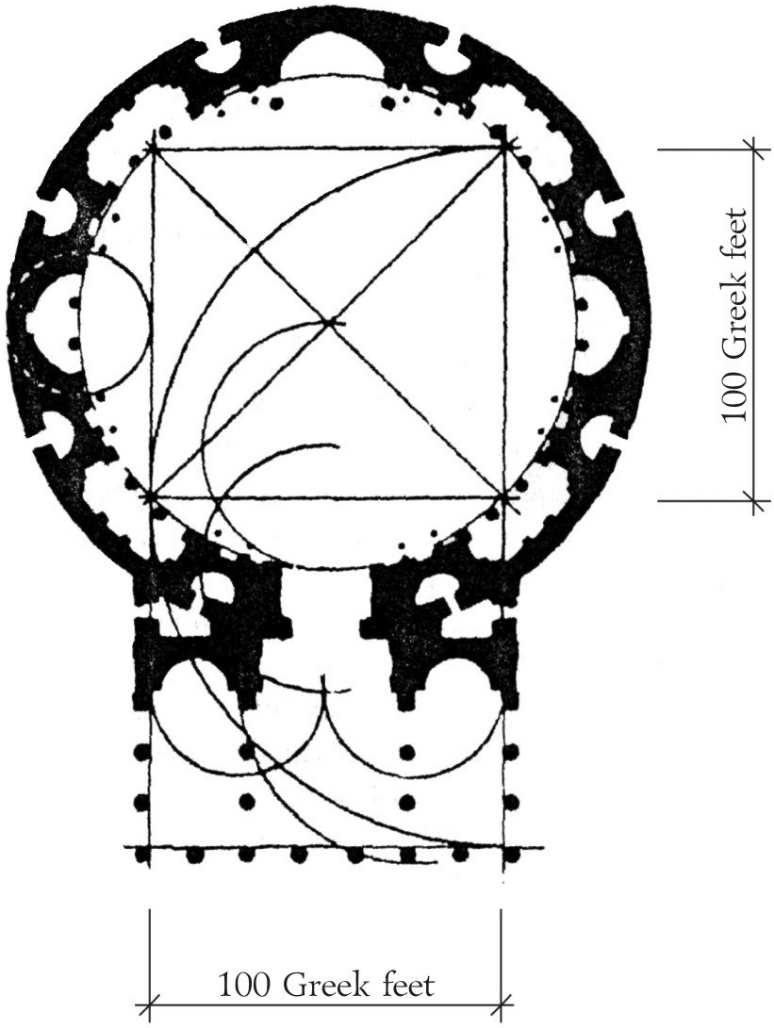
In his treatise “The course of architecture”[3] (1675–1683) he declared the architectural forms of antiquity and the Renaissance to be “eternal and true laws”. He also attempted to find the absolute numerical expression for these laws.

In the middle of the 18th century the Italian researcher Juseppe Erkolani (1672-1759) ventured to develop a method of proportional analysis for some architectural landmark’s forms. In his treatise dedicated to architectural orders[4] (published in Rome, 1744), he gives extremely simple and approximate results of the module and geometric analysis of some architectural landmarks in Rome including the Pantheon (pic. 3).



Pic.3. The scheme of proportional analysis of the Pantheon in cut according to Ercolani.

Two centuries later Russian researcher K. N. Afanasiev (Experience of the proportional analysis) also tried to solve this problem. He made a proportional analysis of the Pantheon basic forms[5] basing on size of the so-called Greek foot (0,308m). According to his reconstruction the internal surface of the Rotunda appeared to be built on a circle which was outlined round a square with the side of 100 Greek feet (pic. 4).



Pic.4. Metrological interpretation of sizes of the Pantheon plan according to K. Afanasiev.

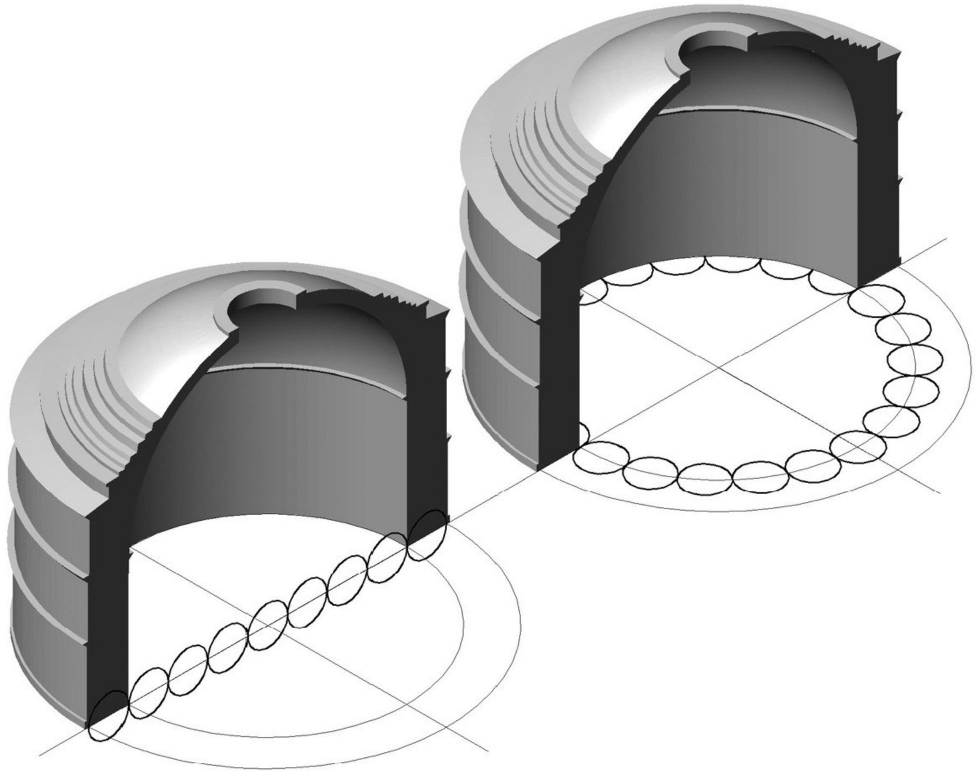
A different metrological basis was used in William McDonald's work[6]. His analysis of the Pantheon forms is based on the size of Roman foot which is 0,296 m, and according to the historic and metrological data, this size corresponds more accurate to this landmark. Unlike Afanasiev, McDonald makes an analysis of only one size — diameter of the dome. This size taken on axes of the big altar columns, is equal to

44,4 m, and is interpreted by him as 150 Roman feet.

Recently, English researcher Mark Jones[7] has made a new analysis of the Pantheon forms using the same foot. This profound and, certainly, the most substantial at present, work contains an attempt to reconstruct the logic of shaping of the Pantheon in a more detailed way. The serious fault of his work, however, is that he does not define precisely such a key size, as the distance between axes of the Pantheon big altars. Reconstructing this size as "round" quantity of 60 Roman feet in a notional ideal, he actually equates it to 59 Roman feet, while uncertainly justifying it by the curvature of the form (pic. 5).

Pic.5. Proportional and metrological interpretation of the Pantheon according to M.Jones.

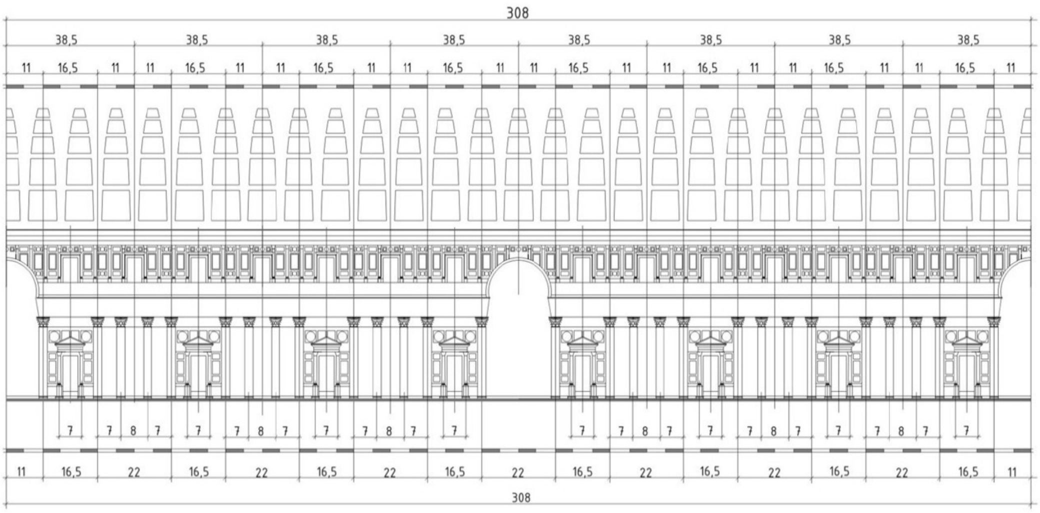
Our attempt to analyze the proportions and the sizes of the Pantheon was based on the automatic technique of the proportional and metrological analysis[8]. The research works of V.Zubov[9] (Architectural and theoretical research and the problems of the its studying”) and John Coulton[10] were used as the theoretical basis. In the works of these authors the purpose of proportional research was defined as means for studying the techniques of proportioning only; which allowed architects to fix design plans and to organize the building process. It is important to notice that V.Zubov and J.Coulton didn't try to search for some global laws of harmony and beauty, but studied the proportion as only an auxiliary tool. As for mathematical aspect, the essence of the technique is in the automatic selection of the module, which is multiple to all sizes of a landmark. As a result of using this technique we got an alternative proportional- metrological interpretation of the Pantheon. It is very close to the theory of M.W.Jones in historical and metrological aspect. However, in our research the size equal to 0,444 m was chosen as a module. This measure is a Roman cubit which corresponds to the size of the Roman foot at a ratio 3/2. Usage of the Roman cubit gave us an opportunity to define the basic regularity in forming of the Pantheon. It appears that the thickness of the Rotunda wall (6,2m) which is equal to 14 Roman cubits, is proportionate to one seventh of internal diameter of the Rotunda (43,57m) which is equal to 98 Roman cubits. Therefore, it is possible to presume that the thickness of the Rotunda wall was used as a certain large module which in its turn served for marking the rotunda forms. The multiplicity of diameter to seven parts, gives us an opportunity to presume that integer fraction 22/7 which in the ancient time, as it’s widely known, served as a very exact substitute of the irrational number «pi» , was also used when rotunda plan was designed (pic. 6).



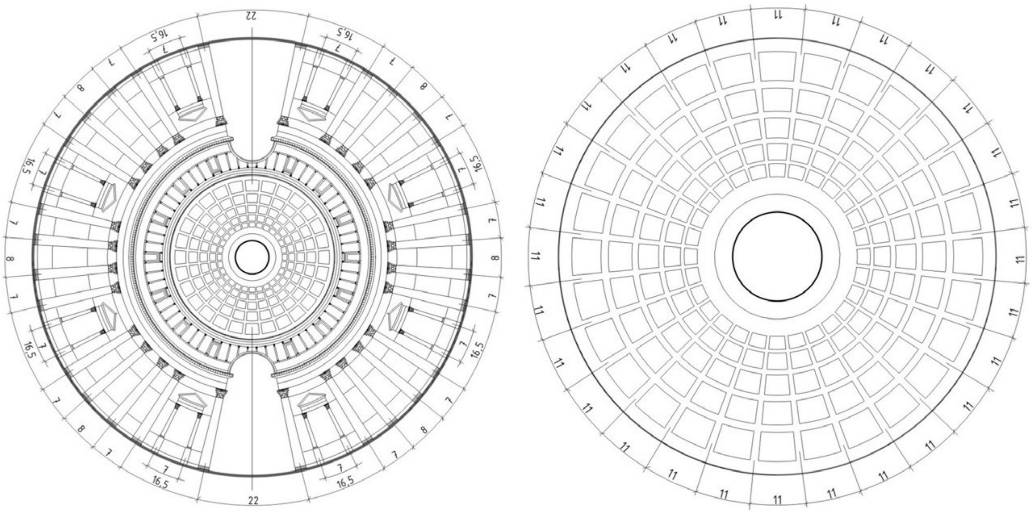
Pic.6. The scheme of reconstruction of a proportional idea of the rotunda plan.

Let's analyze now the sizes of elements compiling the internal length of the rotunda. At first we consider dome elements. The dome consists of caissons which divide the dome into 28 equal parts. According to Desgodet surveys the distance between axes of caissons in the foundation of the dome is equal to 15 Parisian feet (0,3248x15=4,87m) which exactly corresponds to 11 Roman cubits. Therefore, the full length of the dome foundation appears to be equal to 308 Roman cubits (28x11=308). Comparing this size to the diameter of the rotunda, we get correlation 22/7 (308/98=22/7), which is identical to the number «Pi».

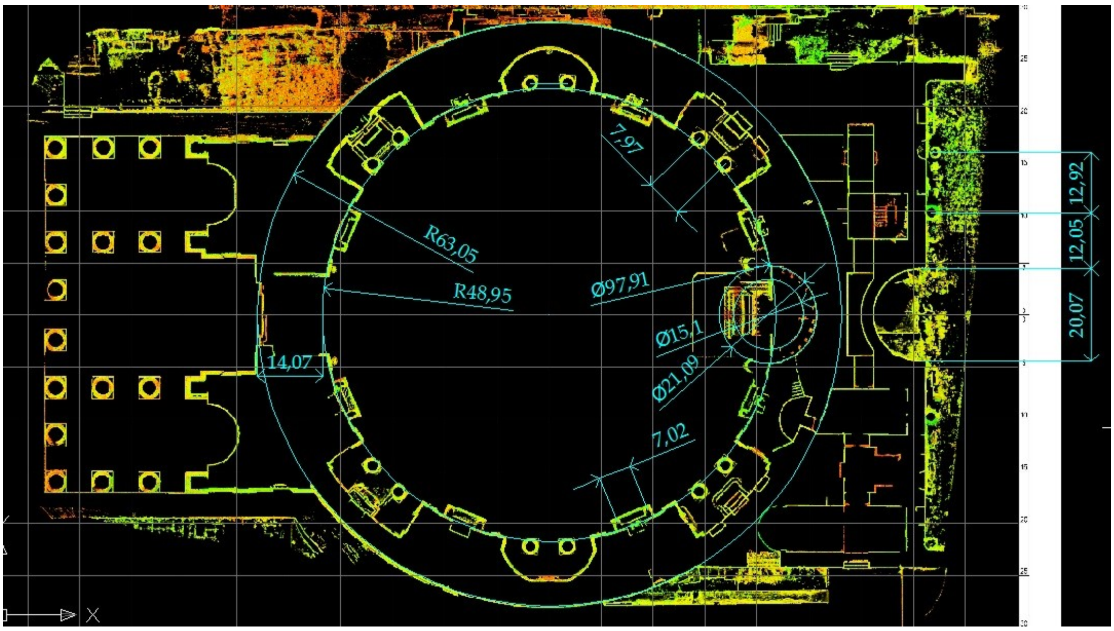
Now we investigate the sizes of the bottom part of the rotunda. It is divided into 8 equal parts. After necessary calculations every such part appears to be equal to 38 ½ of the Roman cubit. Let's re-check this calculation. First of all lot’s, notice that the bottom diameter of the small altar columns is equal to one Roman cubit (0,444m) precisely. Secondly, the bottom diameter of the big altar columns is equal to 2 1/2 Roman cubits (1,11m). The central intercolumniation of the big altar columns is equal to 2,436m, that is 5 ½ of the Roman cubit. Adding it to the measurement of the columns diameters (2 ½ + 5 ½ = 8) the total equals to 8 Roman cubits. Intercolumniation of the side columns of the big altars is one cubit less then a central one, which gives the interaxal size of 7 Roman cubits between side columns. Therefore, the distance between axes of the side columns of the big altars appears to be equal to 22 Roman cubits that corresponds to two steps of caissons of the dome. Herein, interaxal distance between the big columns which gives a space for the small altars, is equal to 16 ½ Roman cubits and this is one and a half time bigger than a step of the caissons of the dome (pic. 7, pic. 8, pic.9, pic.10).



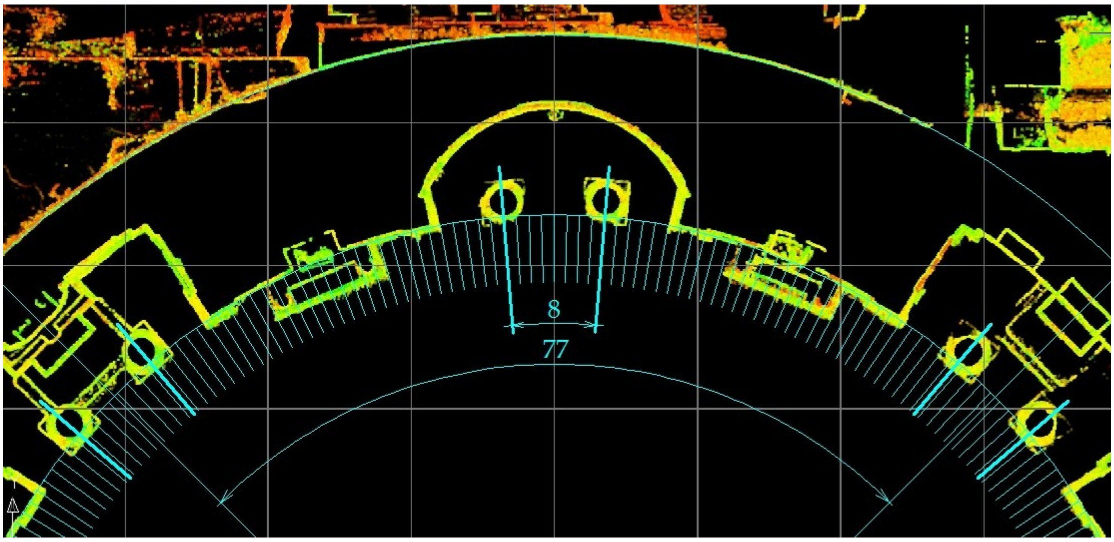
Pic.7. Perspective drawings of the Pantheon interior. Reconstruction of the design sizes. The module is Roman cubit(0,444m).



Pic.8. Unrolling plan of the Pantheon inner surface. Reconstruction of design sizes the module is Roman cubit(0,444m).



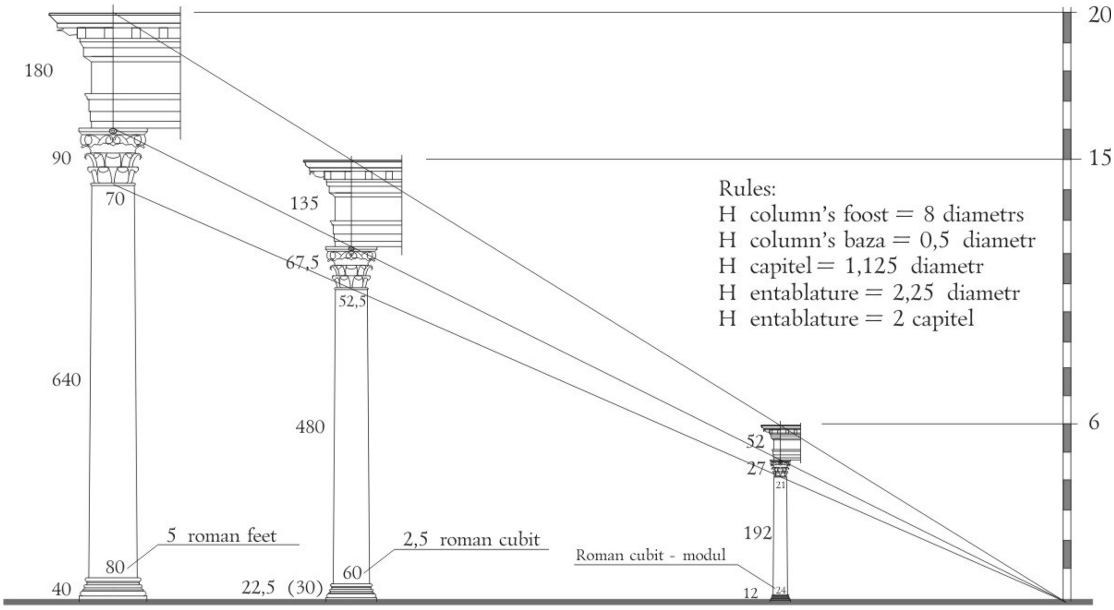
Pic.9. Dimensions of the Pantheon plan in Roman cubit (Analysis of laser measurements from http://www.digitalpantheon.ch/).



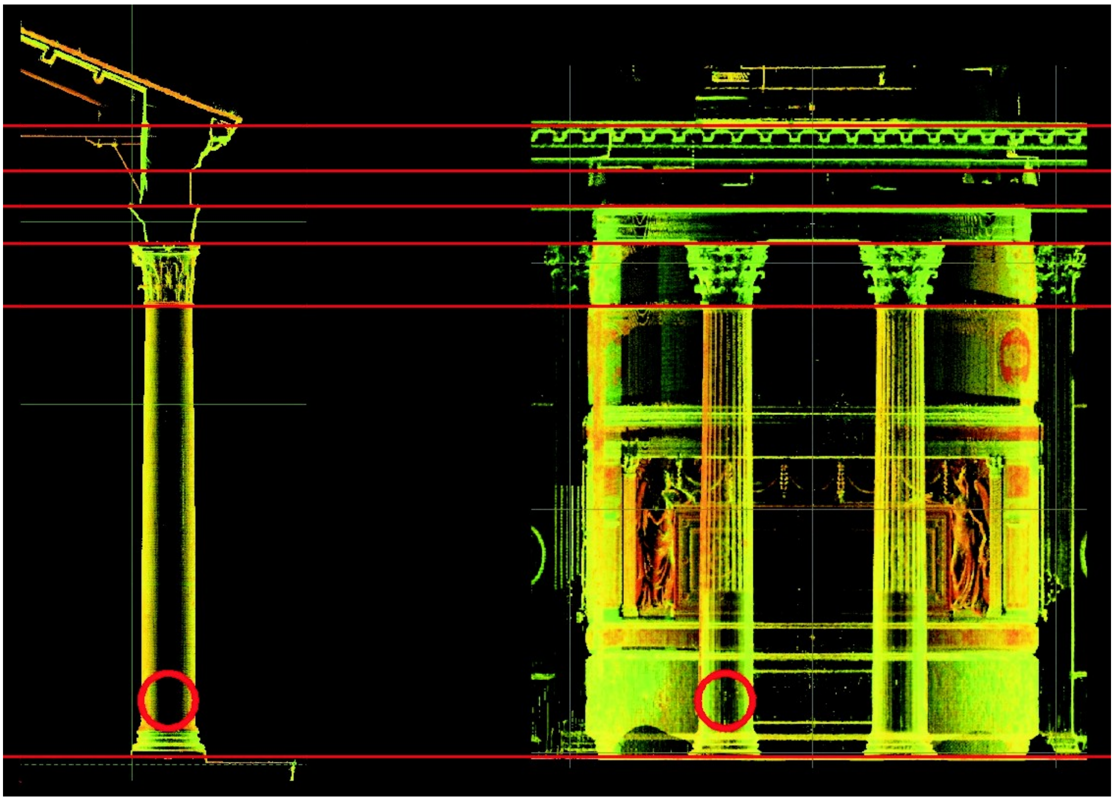
Pic.10. Dimensions of the Pantheon plan fragment in Roman cubit (Analysis of laser measurements from http://www.digitalpantheon.ch/).

It is necessary to highlight one interesting metrological paradox. Desgodet, while measuring details of the Pantheon big altar, used the size of a "part" equal to 1/60 of the column bottom diameter (1,11 m according to the surveys). As it is equal to 2 ½ of the Roman cubit, it corresponds to 60 dactyls (2 ½ x 24 = 60). Therefore, Desgodet's "part" is equal to the size of a Roman «dactyl" and he, although unconsciously, used ancient Roman linear measures for his surveys.

We undertook the detailed measurement of the Pantheon forms in Roman cubits and dactyls (1/24 of a cubit) which gave us an opportunity to find out one very intriguing fact. In the interior, proportions of the portico orders and of big and small altars appeared absolutely identical (pic. 11, pic. 12).



Pic.11. The scheme of the Pantheon orders' similarity. The measurement is Roman dactyl (0,0185m=1/24 of Roman cubit).



Pic.12. The scheme of the Pantheon orders' similarity (Analysis of laser measurements from http://www.digitalpantheon.ch/).

Identification of analogies in the Pantheon orders proportions allowed us to hypothetically define a number of rules of their constructions:

- The height of a trunk of a column is equal to 8 bottom diameters of the column;
- The height of the base of the column is equal to 1/2 of the bottom diameter of the column;
- The height of the capital is equal to 9/8 of the bottom diameter of the column;
- The height of the entablature is equal to 9/4 of the bottom diameter of the column;
- The height of the entablature is two times more than the height of the capital.

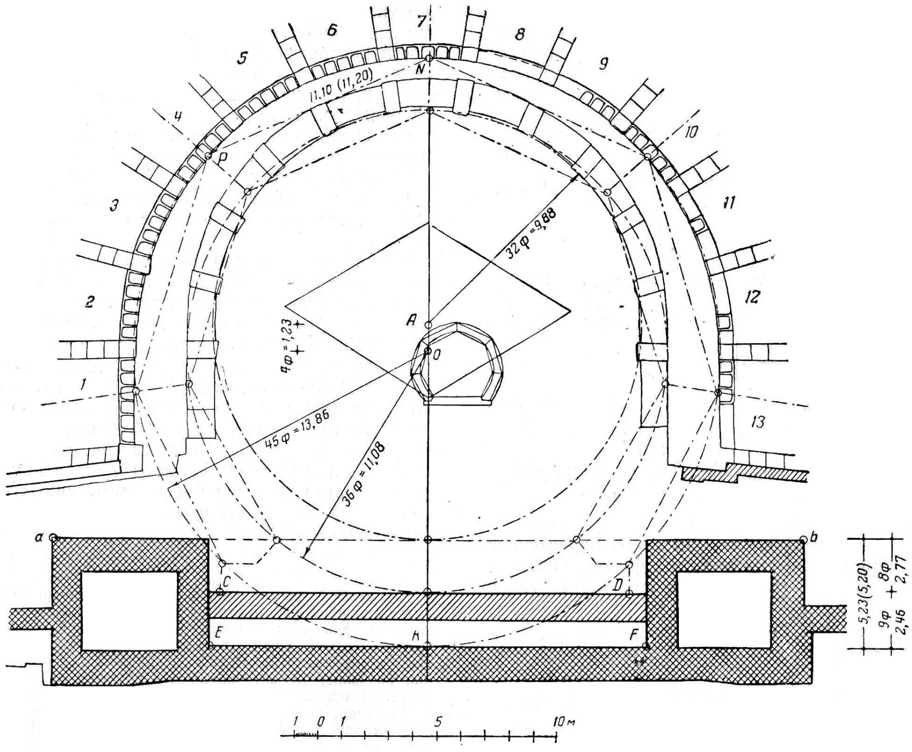
It is possible to give the following interpretation to the fact of full similarity of three orders which have different absolute sizes. Firstly, we can assume that the portico was attached, or designed, at the same time as the rotunda was. Secondly, similarity of different in size orders, shows that the architect of the Pantheon did not know about the treatise of Vitruvius who suggested that proportions of the columns and architrave beams should change depending on the absolute sizes of the order system. This implies that recommendations of Vitruvius to consider the scale factor were ignored.

However, it is necessary to note, that even Alberti who had studied the treatise of Vitruvius in details, did not understand this recommendation under V.Zybov statement ("Alberti's architectural theory")[11]. Besides, it is possible to assume that the contents of the treatise of Vitruvius were not absolutely clear for architects of that time as it had been written in the old Roman language, with numerous inclusions of the Greek texts taken from the other more ancient sources.

The results of the analysis of other known constructions which are circular in plan, additionally confirms this interpretation of the Pantheon sizes. In particular, we can mention some Ancient Greek theatres, such as Theatre of Dionysus in Athens and the Theater in Piraeus, the plans of which were designed with the division of half of the circle into 11 parts.

It is necessary to notice that our metrological reconstruction of the plan of the theatre of Dionysus in Athens, completely support's the given theory. Thus, according to Fichter's measurements[12], the radius of the basic circle of the theatre is 13,7m which can be interpreted as 42 Doric feet consisting of 0,326m. Therefore, the length of such circle turns out to be equal to 132 Doric feet. Thus, the initial width of one sector received by division of a circle into 22 parts, turns out to be equal to 6 Doric feet. The presence of this very measure in the forms of the theatre is proved by the fact that another landmarks of Acropolis, where the theatre is situated, were designed with the use of Doric linear measures. This is according to «The Estimate of the Erechtheum building».

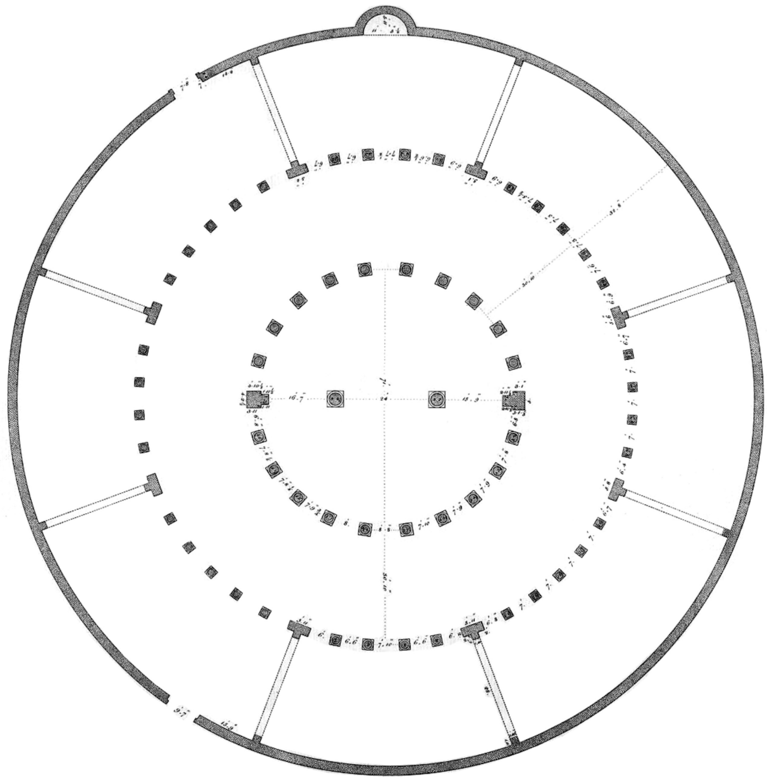
Our analysis of the antique drawing made on the slabs has shown that this drawing was executed at a scale of 1/6. The drawing radius is equal to 2, 29m that makes 7 Doric feet. One foot of the drawing corresponds to six feet of the real object (pic. 13).



Pic.13. Measure drawing of a plan of the theatre of Dionysus in Athens combined with the ancient drawing according to V.E. Bykov reconstruction.

Most likely, in this case we deal with the first fact of use of the scale drawing put on the surface of construction floor marble slabs[12]. The execution of the first scale drawings on paper should most likely be attributed to Andrea Palladio. Our detailed analysis of the engravings[13] with Pantheon drawings has shown that they are executed in such scales as 1:20, 1:80, 1:160 and 1:320.

The above mentioned extensive research of Desgodet contains the measurements of several buildings, circular in plan apart from Pantheon. It is extremely significant that in the plans of the circular Temple of Faunus in Rome, the circular interior space of the temple is divided by columns into 22 and 44 parts what indicates that 22/7 fraction was used (pic. 14).



Pic.14. The plan of a round temple Fauna in Rome which circle is divided by columns into 44 sectors (on measurements of Desgodet)

The presence of the multiple of seven in the sizes of the circular in plan architectural monuments, can be found in different periods and in different countries. For example, the size of the dome of Hagia Sophia in Constantinople, 30,9 m, was equated to 100 Greek feet in K.N.Afanasyev's reconstruction. However it is possible to express this size through the Roman linear measures. It could be equated to 105 Roman feet or 70 Roman cubits. In this case the diameter turns out to be multiple to 7 again and the internal length of the dome, according to calculation, turns out to be equal to 220 Roman cubits (330 Roman feet). The diameter of side exedras of the temple is 14 Roman cubits. This also points to the fact that the use of 22/7 fraction while laying their forms out was possible.

In a number of researches the diameter of the Pantheon is often compared to the diameter of the dome of St. Peter's Cathedral in Rome. It is noted that they are very much alike. According to reference data diameter of St. Peter's Cathedral is 41,47m. As this cathedral, according to our research, was also designed in Roman linear measures, it is possible to suggest that its diameter is equal to 140 Roman feet. This size is also multiple to 7. If we use integer value of "Pi" - 22/7, we will get the internal length of the drum to be equal to 440 Roman feet. There are considerable quantity of examples of the metrological multiplicity to 7 in the diameters of the church domes in Russian architecture:

Architecture monument	The size of the central nave (diameter of the main dome) in metres	Metrological interpretation
Cathedral of the Dormition of Kiev Pechersk Lavra (1073-1076)	8,65	28 Roman cubits
Saint Sophia Cathedral in Kiev (the 11-century)	7,75	17,5 Roman cubits
The Cathedral of St. Sophia in Veliky Novgorod (1045-1050)	6,2	21 Roman feet or 14 Roman cubits
Dormition Cathedral in Vladimir (1158-1160)	6,5	21 Byzantian feet

The Church of the Nerl River (1158)	3,1	7 Roman cubits
The Cathedral of the Dormition on Cathedral Square in the Moscow Kremlin (1475-1479)	8,65	28 Roman feet
The Cathedral of Christ the Saviour in Moscow (1839-1883)	On scale of the design drawing	35 arshins

Conclusion

The results of this research show that arithmetic fraction 22/7 as the analogue of "Pi", was widely used in designing of circular elements in constructions. The practical necessity in designing of circular forms forced the architects who had neither computers, nor calculators, to use accessible elementary mathematical means for circumference plotting.

Notes

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13. *Радзюкевич А.В.* Пропорционально-метрологические и масштабные особенности чертежей А.Палладио. [Electronic resource]: Radzyukevich A. Proportional and metrological and scale patterns of the A.Palladio drawings.
http://www.a3d.ru/architecture/stat/234

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